DIFFER FROM THY NEIGHBOUR:
PRODUCT POSITIONING IN MULTIDIMENSIONAL MARKETS

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Abstract

The presentation extends the model of spatial competition proposed by Hotelling (1929) to *n* dimensional commodity spaces, with the assumption of equal product prices. Shops seek to position their product catchment areas to minimize competition, i.e., overlap with neighboring supply areas. I address two types of *n*-dimensional filling up of the commodity space with supply areas around shops. Cubic cell arrangements are easy to build up in any *n*. Sphere-like catchment areas cells allow for more shops per space unit, but the building up of such an arrangement is more problematic.

The paper of Harold Hotelling (1929) serves as a benchmark for economists in spatial competition research. He determined the catchment areas of two shops that sell the same product at two different locations along a finite line (e.g., the shopping street in a town). The model has been generalized in several aspects by the release of some of its simplifying assumptions (see a selection of articles at Dean, Leahy and McKee 1970, and an overview at Darnell 1990). Salop (1979) considered more than two firms that position along a bounded one dimensional space, a ring of unity circumference.

This presentation considers possibilities how shop catchment areas can look like in *n*-dimensional commodity spaces. In a two dimensional approach to the catchment area problem, shops search locations along a surface. The research lines set out by Christaller (1933) and Lösch (1940) studied geographic firm positioning on a plane (Christaller 1966; Lösch 1967Funck and Kuklinski 1986; Beckmann and Puu 1990,; Nooteboom 1993). In a geographic context, higher dimensional space partitionings are irrelevant. But Hotelling mentioned in his paper the "sweetness of the cider" as a third dimension, suggesting the possibility of adding axes that stand for commodity characteristics. Our presentation takes this approach, considering that the *n*-space is spanned by *n* independent product characteristics taken into account by the customers (Lancaster 1966).

Hotelling's Approach

In Hotelling's original setting, demand was exogenously given, inelastic and uniformly distributed along the line. Price differentiation between customers is excluded. Production costs were equal and can be considered to be zero without an effect on model generality. Each shop offers a single product. Customers choose between the two products only on the ground of price and transportation cost that stands for the buyer's inconvenience because of a remote shop location. If transportation cost is a linear function of distance, then t he following equation holds for a customers that stays at the division line between the two shops’ sales areas:

$$(1) \qquad p_1 + c \cdot l_1 = p_2 + c \cdot l_2$$

where p_i is the price of shop i, l_i is the distance between shop i and the customer, and c is the transportation cost for unit distance. If the shops take position symmetrically along the line, then equal sales areas with equal prices obtain as equilibrium solution.

In *n*-dimensional commodity spaces, products and customer tastes are represented by points in this space. The transportation costs stand for the customer's inconvenience because of the mismatch between her/his taste (ideal point) and the characteristics of the offered product. Equation (1) can be applied to determine catchment area boundaries also in *n*-dimensional commodity spaces and with any number of products in place. But, equation (1) does not guarantee convex catchment areas, if $n > 1$. Writing about model generalization to 2-D, Hotelling mentions that the two shop cells are separated by a hyperbolic line on a plane if transportation costs are linear. However, if the prices are equal, then the convexity problem goes away. Therefore from now on, the presentation studies the *n*-dimensional generalization of the Hotelling problem assuming equal prices. In that case, each cell border is a straight line that stands midway between shop positions (see Slide).[1]

With the above mentioned assumptions in place, shops' market positions are quite similar, so large dissimilarities in supply area sizes do not lead to stable market partitionings. As customer satisfaction lowers with the product-customer distance, large catchment areas may attract new entries at the border regions. How does a "good" market partition with similar catchment area sizes look like in an *n*-dimensional Lancasterian commodity space? A partition is "good" if the supply areas (monopoly market zones around firms) are of similar in size, and the cells diameter is similar in every direction ("sphere-like cells"), so the arrangement has no extra week points that attract firm entry. Moreover, a good arrangement should be relatively easy to build up.

Looking for a solution to this problem, we resort to the sphere packing problem of geometry (Conway and Sloane 1998), where the central question is: "How can the *n*-space packed up with non-overlapping spheres of equal radius in the "densest " way? Packing density (Δ) is measured by the ratio of space occupied by spheres to total space volume. The sphere centers in dense sphere arrangements will point of the shop locations with which in place the supply area volumes and shapes become similar.
The presentation studies two ways of filling up the space with spheres:

(i) <u>Product locations form a system of *n*-dimensional cubes</u> (sees)
In this setting, the supply areas around the touching *n*-spheres are *n*-cubes. The entailing cell arrangements builds up easily in any dimensions. Firms have to follow a strategy that

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[1] This brings to the quantizer problem of data compression, and the obtaining catchment areas turn out to be the Voronoi cells known from coding theory (Conway and Sloane 1998, p. 33).

combines product imitation and product differentiation: look for an empty location around an existing sphere (imitation), and differentiate your product along one dimension (with $2r$, where r is radius). But the problem with this arrangement cubic configurations become increasingly loose: the cube's center-vertix distance (the maximal misfit between product and buyer) is proportional to $\sqrt{n}$, and thus it increases towards infinity with n. This setting invites competitor entry in higher dimensions. From space dimension $n \geq 4$, the center-vertix distance of the cubes becomes at least $2r$. That is, an extra supply area of the same r radius can be inserted to every vertex! This invites a strong wave of entry. The packing density of the arrangement can be doubled at $n = 4$, if the entry condition is that the entrant has to secure a catchment area of the same diameter as incumbents.

(ii) Dense market packings with spherical cells.

Though the sphere packing theory usually cannot provide the densest sphere arrangement for higher dimensions, it specifies very good arrangements that approximate the optimum. See the known densest packings and their properties in the slides. Markets densely filled up with spherically symmetric product areas are exempt of the problem of increasing misfit (Péli, Gábor and Nooteboom, 1999). However, it is difficult to find and to build up a dense arrangement. In stable markets, selection processes may press the system towards such settings (Carroll and Hannan 2000).

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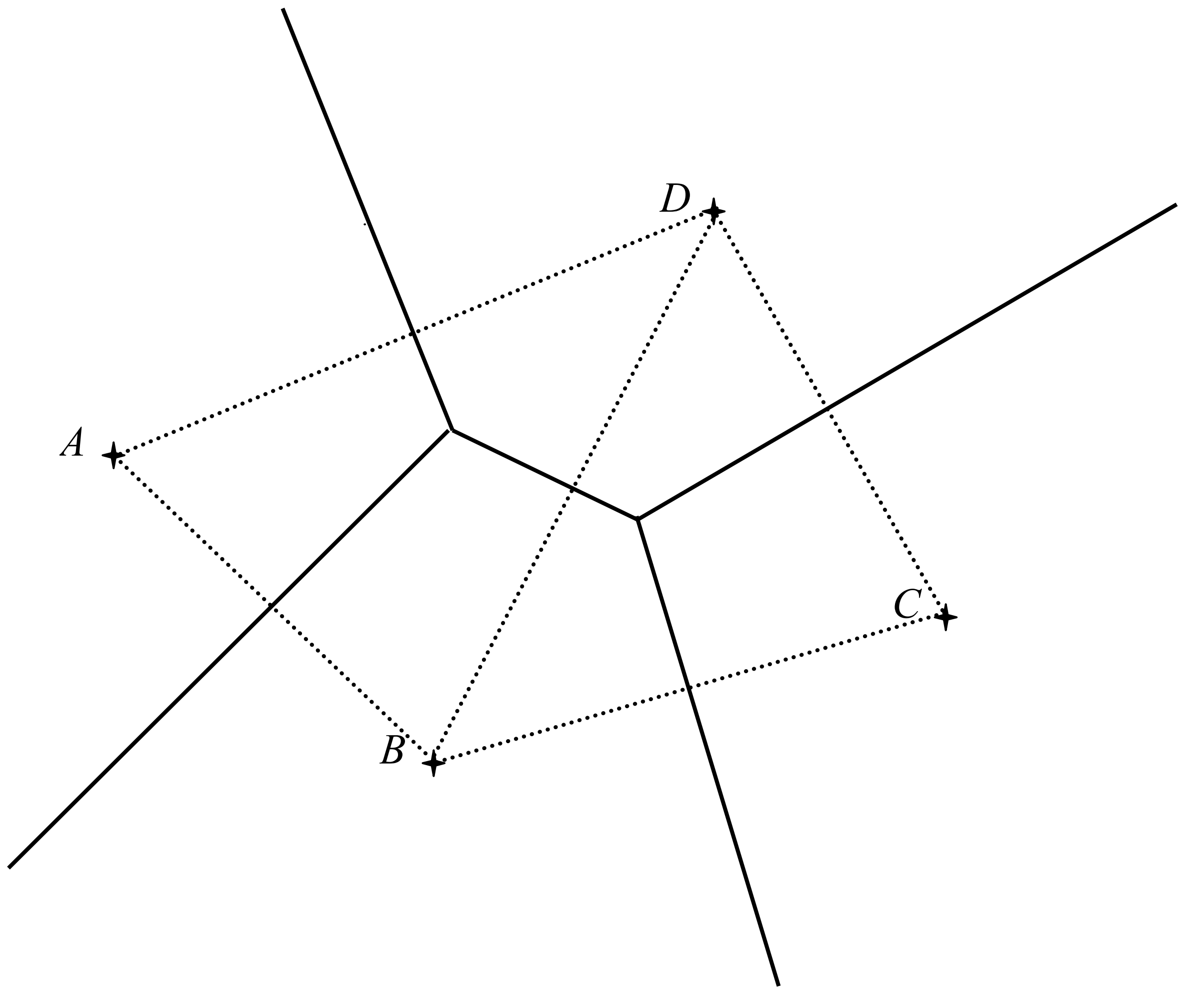
SLIDES



A Two-Dimensional Quantizer:

A,B,C, D are shop positions,

I. Niche Positioning in the *n*-Space of Resources

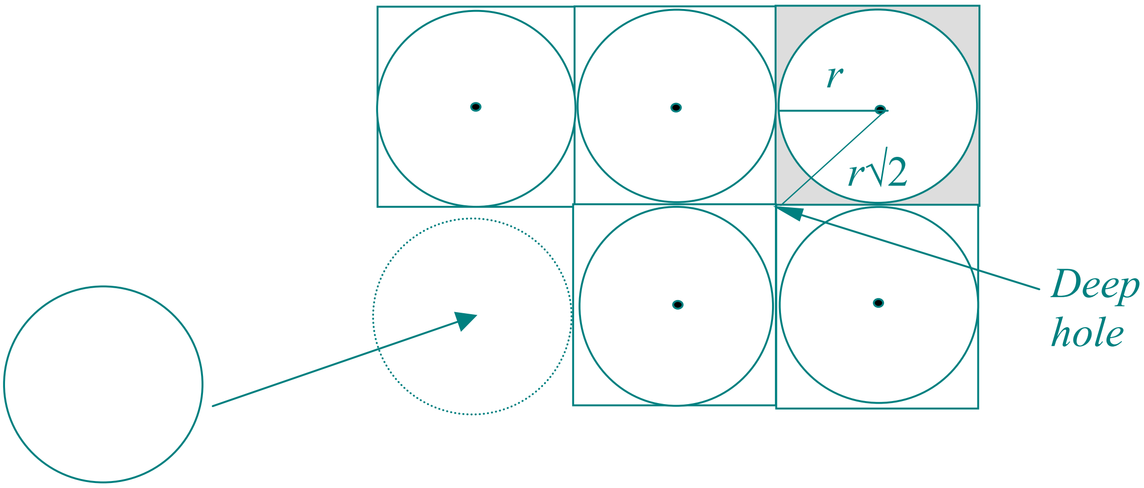
The goal is to place a maximal number of non-competing organizations into the resource space (avoiding niche overlap or keeping it low).

Two kinds of spatial arrangements are studied:

- Simple niche configurations
- "Economic" (denst) niche configurations.

Simple Niche Arrangements

Niche centers form a lattice of n-dimensional cubes.



Advantages **-** Builds up simply.

- Easily generalizes to any dimensions.

Disadvantage. Too "loose".

Deep hole distance goes to infinity with n: $\mathbf{r\sqrt{n}}$

If $n = 4$, then the deep hole size is: $\mathbf{r\sqrt{4} = 2r}$

A new organization of the same or bigger niche size

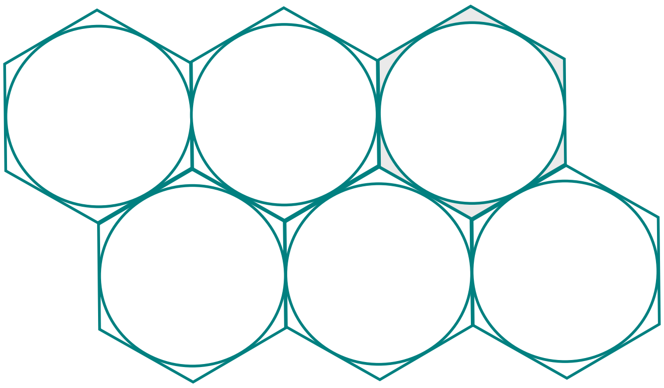
can be inserted into each deep hole if $n \geq 4$.

Dense Packings

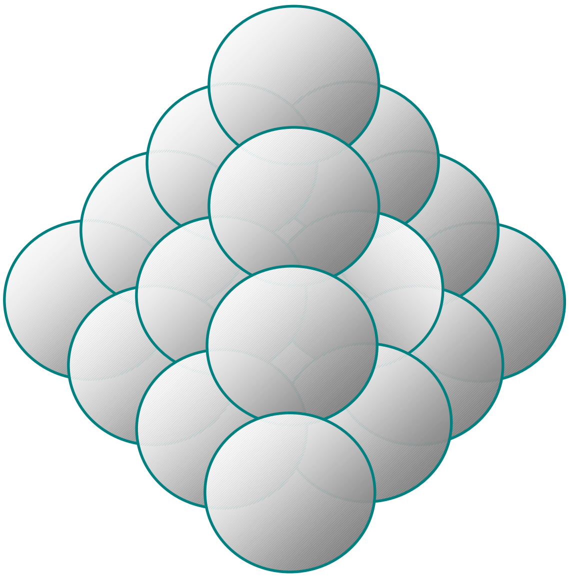
The sphere packing problem:

How to place the highest number of non-overlapping unity spheres into the *n*-space?

2D: Max. packing density
$\Delta = 0{,}91$



3D: Max. packing density
$\Delta = 0{,}74$



- **Density rapidly converges to zero with *n* in the known best packings.**

- **The number of immediate neighbors** (kissing number, τ) **goes to infinity with *n*.**

n	**Density Δ** best known packing	**Density Δ** cubic center arrangement	**Kissing Number** τ
1	1	1	2
2	0.90690	0.7854	6
3	0.74048	0.5236	12
4	0.61685	0.3084	24
5	0.46526	0.1645	40
6	0.37295	0.0807	72
7	0.29530	0.0369	126
8	0.25367	0,0159	240
9	0.14577	0,0064	272
10	0.09962	0,0025	372